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# Connectivity of Swarm Robot Networks for Communication Range and the Number of Robots Based on Percolation Theory

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***Abstract*— One approach in swarm robotics (SR) is homogeneous system which is embedded with sensing, computing, mobile and communication components. This is identified with mobile wireless sensor networks (WSNs). For some SR tasks, robots need to collect information from the environment and share their data with each other. Due to the multi-hop transmission of WSNs, robots in such networks can communicate with each other via intermediate relay robots. Therefore, it is important to take connectivity of the network into account. This study investigates communication range and the number of robots required for a SR network to achieve connectivity based on percolation theory.**

## I. INTRODUCTION

Swarm Robotics (SR) [1][2] have attracted much research interest in recent years. Generally, the tasks in SR are difficult or inefficient for a single robot to cope with as those in *multi-robot* systems. Sahin [3] enumerated several criteria[1] for distinguishing swarm robotics:

- autonomy: Each robot should be physically embodied and situated.
- redundancy: Group sizes accepted as swarms is 10 to 20.
- scalability: SR system should be able to operate under a wide range of group sizes.
- simplicity: Each robot should employ cheap design, that is, the structure of a robot would be simpler and the cost for it would be cheap.
- homogeneity: SR system should be composed of homogeneous individuals. This enhances the above 2nd and 3rd criterion.

Following the last criterion, homogeneous controllers for individuals are desirable for SR systems. This approach does not assume the existence of an explicit leader in the system due to the above criteria. This results in that a collective behavior emerges from the local interactions among robots and between the robots and the environment. Therefore, SR systems are required for that individuals show various behaviors although the individuals are homogeneous.

In SR, the typically control tasks requiring distributed collective strategies have been coped with, navigation, aggregation, formation and transport. This paper supposes we copes with a navigation problem. In this control task, several robots can communicate with each other via wireless sensor networks (WSN)[4][5] due to the multi-hop transmission to achieve collective exploration. As soon as a robot detects a target, the information is sent from the robot to the base station via intermediate relay robots. Therefore, all robots should be "connected" to the base station via WSN.

This paper investigates communication range and the number of robots required for a wireless sensor network composed of swarm robots (which is called "SR network" in the remainder of this paper) to achieve *connectivity* base on the percolation theory[6]. The paper is organized as follows. The next section shortly introduces percolation theory. Section III conducts computer simulations. Section IV investigates the validity of the results in a robotics control task and discusses utilization of the results. Conclusions are given in the last section.

## II. PERCOLATION

This section shortly introduces percolation theory.

A site is randomly arranged with probability $p$ on lattice points of a square lattice. Sites become adjacent to each other when $p$ becomes large from 0. Above a critical value of $p$, clusters of big size first form where any sites between opposite boundaries are interconnected, that is, sites *percolate*. Such a process in that clusters form is called, "*site percolation*". For another process, lines are randomly add between neighbouring lattice sites until clusters form. This is called, "*bond percolation*". In this way, percolation theory discusses connection between elements composing a cluster and characteristic emerged from a set of connected elements [6].

For an infinite set of sites, it has been reported that a critical value at which an infinite large cluster forms depend on the shape of a lattice. In some cases of two- or three-dimensional lattice whose shape is simple, a critical value can be calculated explicitly. In the rest of cases, such a critical value cannot be calculated exactly so that this value is estimated by computer simulations. The above explanation is based on a lattice, that is, discrete region.

As those studies in a lattice, several researchers investigated percolation in continuous space [7][8]. In continuous space, the concept, "neighbouring lattice sites", does not exist. Therefore, the bonding criterion must be defined. Fig. 1 shows the bonding criteria in two-dimensional continuous space: Two sites are bonded if those are within each other's circle of radius $R$, that is, $d \leq R$, where $d$ is the distance between two sites. In the reference [8], the authors investigated percolation on the two-dimensional continuous space. $N$ points are uniformly distributed in a unit square in Fig. 2. About each point, a circle of radius $R$ is drawn. Two

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[1]Sahin [3] claimed that these criteria should be used as a measure of the degree of SR in a particular study.

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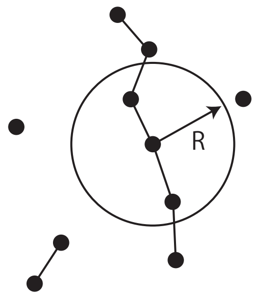



Fig. 1. Illustration of the bonding criteria for two-dimensional continuous space

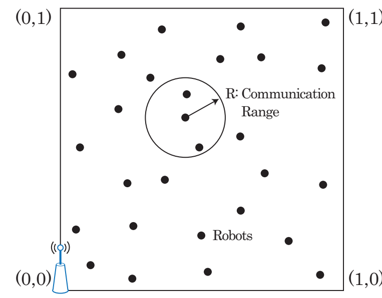



Fig. 4. Experimental setup for two-dimensional continuous space

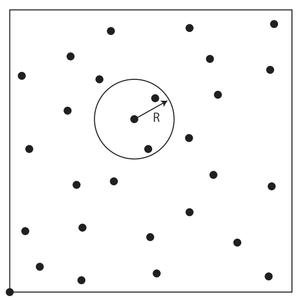



Fig. 2. Two-dimensional continuous space

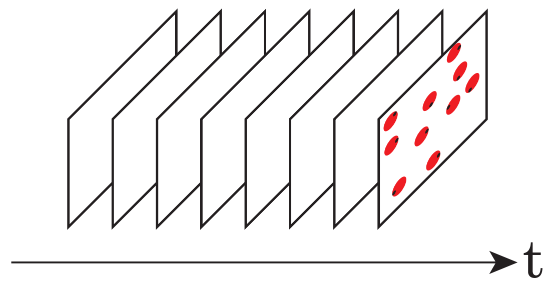
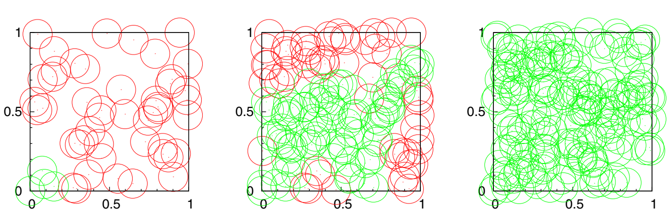



Fig. 5. Snapshots of robots navigation

points are considered to be connected when their distance is less than $R$ (Fig. 1). For this setting, you can imagine the following situation: The value of $R$ is increased with the constant $N$ or the value of $N$ is increased with the constant $R$ until the clusters of big size are observable between opposite boundaries. The phenomenon at these critical values is also called, *percolation*.

Fig.3 illustrates percolation in continuum region[2]. The radius of a circle shows $R$. The center point in the green circle is connected via other center points from the origin while the point in the red circle is unconnected. The value of $N$ is increased while $R$ is fixed at $0.1$. When $N = 50$, a few connections from the origin appear (Fig.3(a)). When $N = 150$, many connections appear although the connections are not observable on the far side of the square (Fig.3(b)). When $N = 200$, the connections are observable throughout the square so that a cluster forms (Fig.3(c)).

[2]This situation described here is not the same as the original one in [8] but is related to our setting of this study (the details are described later).




(a) $N = 50$ (b) $N = 150$ (c) $N = 200$

Fig. 3. Percolation in continuous space

## III. COMPUTER SIMULATIONS

### A. Simulation Conditions

In this experiment, we considered the model in two dimensional space in percolation theory as described in the previous section. We uniformly distributed $N$ points in a unit square in Fig. 4. About each point, a circle of radius $R$ is drawn. This corresponds to a SR network as follows; A point $i(i \in \{1, \cdots, N\})$ is an arbitrary robot, The point density, $N$, is the number of robots in a unit square and $R$ is communication range for each robot, where $R$ is set to constant for all the robots because we assume the system in a SR to be homogeneous as described in Section I. The base station locates at the origin, whose communication range is $R$. Mobile robots are supposed to be used in this study. However, the setting described above is static, that is, the positions of the robots are fixed. This is because we consider this setting to be a snapshot during robots' navigation (Fig. 5). According to this setting, we investigated connectivity among the robots and between the robots and the base station. The values of $R$ and $N$ are increased until the connections are observable between the most of those robots and the base station, that is, percolation is detected. Above this critical value, each robot can communicate with the base station via intermediate relay robots. This is a different point from the original model on the two-dimensional continuous space. For this experiment, $R = \{0.1, 0.2, \cdots, 0.9\}$ and $N = \{2, \cdots, 1400\}$. The robots were randomly distributed in a unit square. We conducted $10000$ independent runs for each problem with the parameters ($N$, $R$). All results were averaged over $10000$ runs. These several runs with the random distribution make possible to consider a series of

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*snapshots* to be the behavior of the robots during navigation. So far we have described the common setting in a series of experiments conducted in this study. Particular setting for each experiment will be described in the following part.

## *B. Simulation Results*

*1) Without body:* In the first experiment, we do not consider the body of the robot and treat it as a point. In Fig. 6, the ratio, $P$, of the number of the connected robots directly or indirectly via intermediate relay robots from the base station for each $N$ is shown on a semilogarithmic graph for each $R$. $P$ is sometimes called *connectivity* in the remainder of this paper. $P$ increases with the increase of $N$. $P$ for the extreme large value of $R$ is more than 0.8 even when $N = 2$. We can find the critical value of $N$ at which $P$ converges to 1.0 for each $R$. Table I shows the critical values obtained in this experiment ($r = 0.0$).

*2) With body:* So far we have not considered the body of the robot located at a point. That is, an infinite number of robots can be located in the neighborhood of a robot. In the second experiment, we considered the body of each robot by introducing a circle of radius, $r$ [8], where $r \in \{0.05, 0.075, 0.1\}$ and $r < R$. The reason why the condition $r < R$ is introduced is that the radius which is larger than communication range ($r > R$) is not practical. Here, we conducted the same experiment as the first one after the robots were randomly distributed in a unit square avoiding overlap between robots and the border of the environment. Note that there are cases in which we can not distribute $N$ robots due to the body of the robot.

Fig. 7 show the effect of the body on $P$. The graphs were plotted for each $R$ in Figs 7(a)-7(d) in order to avoid overplotting. Note that the scales of horizontal axis are different for each graph, and that the results for $r = 0.0$ were the same as those in the first experiment (without body). For small values of $R$, there were the cases where the calculation of $P$ was not able to be executed, e.g., $r = 0.075, 0.05$ in Fig. 7(a) and $r = 0.075, 0.1$ in Fig. 7(b). This is because we can not distribute $N$ robots due to the body of them

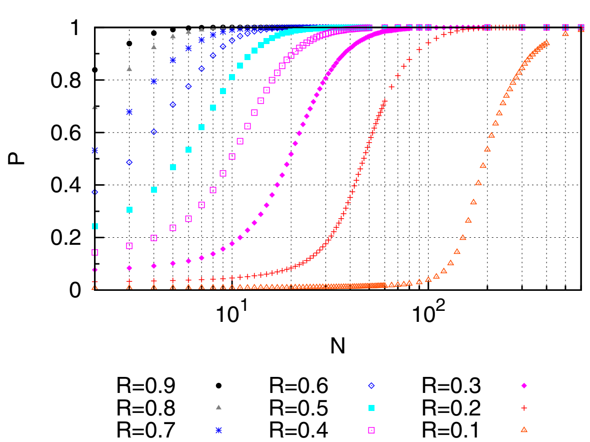



Fig. 6. Connectivity for each communication range

although larger $P$ needs more $N$ for small $R$ as mentioned before. For large $R$, $P$ becomes large when $r$ is large in Fig. 7(d). Addition to this, we can find lower bounds of $P$ for each $N$ when $r = 0$. We observed the same tendencies as this for other values of $R$.

For each $r$, the critical values of $N$ are shown in Table I[3]. When $R$ is more than 0.5, connectivity were observable for less $N$ by introducing the body of each robot which makes robots distribute evenly.

*3) Under uncertainty:* In Section III-B.1 and III-B.2, we employed the simplified communication range model (Fig. 1). In real environment, however, there are several uncertainty, such as noise and interference, scattering, diffraction, and reflection of other transmissions and obstacles [5]. Thus, we consider a probabilistic communication model. However, no general communication model considering uncertainty has not proposed. In this study, we employed the probabilistic communication model in which the communication radius, $R^{'}$, is a random variable following probability distributions in Fig.8.

Two types of distributions were used: uniform distribution and Gaussian distribution. A communication radius, $R^{'}_u$, distributes uniformly between 0 and $R$: $R^{'}_u \in U(0, R)$ (Fig.8(a)) where $R$ is the communication range of a transmitter employed in Section III-B.1 and III-B.2. Another communication radius, $R^{'}_g$, follows $N(\mu, \sigma^2)$ where $\mu$ is average and $\sigma$ is standard deviation. We set $\mu$ at $\frac{R}{2}$ and $\sigma$ at $\frac{R}{6}$, respectively. Following $N(\frac{R}{2}, (\frac{R}{6})^2)$, $R^{'}_g$ falls within the three $\sigma$ range, $\mu - 3\sigma \leq X \leq \mu + 3\sigma$, at a probability of 0.9999 (Fig.8(b)). A common features between these probabilistic communication radiuses are that the minimum value is 0 and the maximum value is effectively $R$. This corresponds to the simplified communication range in Fig.1.

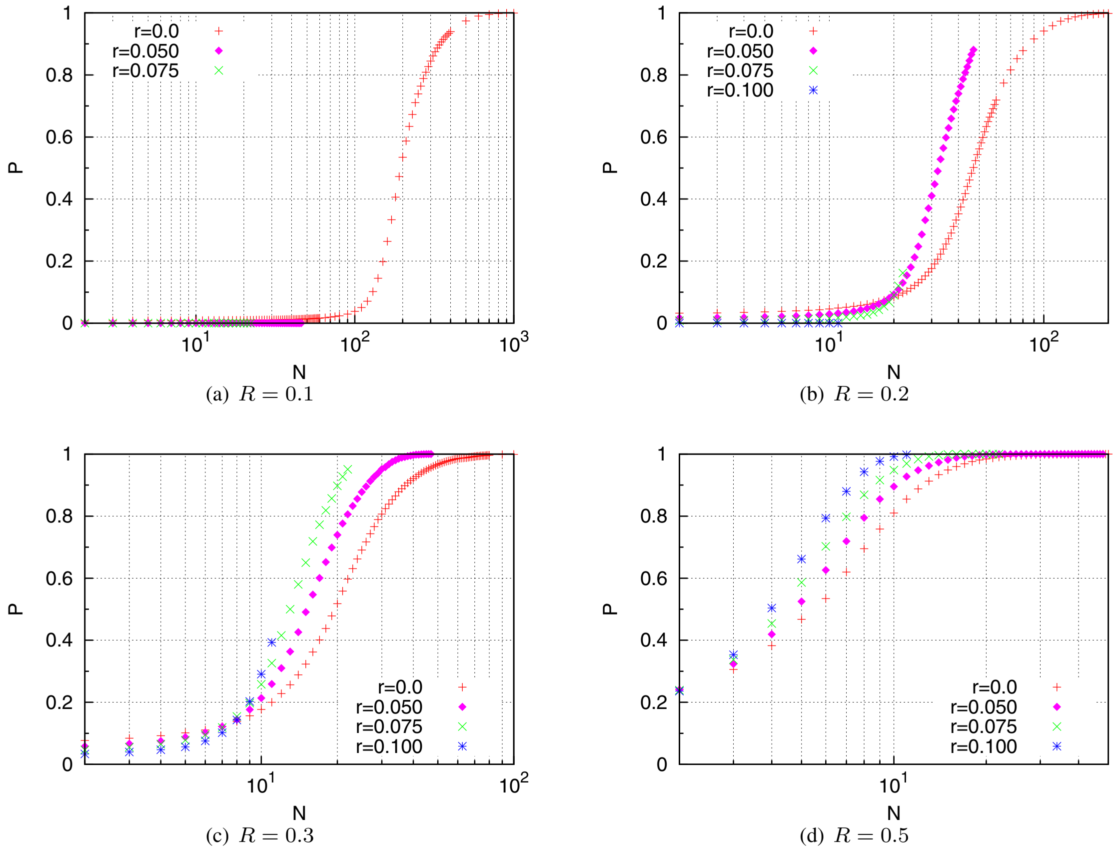
According to the above distributions, $R^{'}_i$ and $R^{'}_j$ are generated for the $i$-th and $j$-th robot, where $^{\exists}i, j \in \{1, \ldots, N\}, i \neq j$. Two sites are bonded, that is, two robots can communicate with each other when $d \leq \min\{R^{'}_i, R^{'}_j\}$, where $d$ is the distance between two robots. For this experiment, $R = \{0.1, 0.2, \cdots, 1.0, \cdots, 5.0, 10.0, 20.0\}$, where some $R$ are larger than those in Section III-B.1 and III-B.2. We conducted 100000 independent runs for each problem with the parameters ($N$, $R$).

[3]'x' shows nonexecution of the calculation and *–* shows no setup in Table I.

TABLE I

CRITICAL VALUES OF $N$ FOR EACH $R$ AND $r$, ABOVE WHICH A NETWORK IS FULLY CONNECTED.

| | $R$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $r$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
| 0 | 1388 | 341 | 163 | 83 | 50 | 34 | 20 | 16 | 9 |
| 0.050 | x | x | 47 | 36 | 26 | 19 | 17 | 12 | 7 |
| 0.075 | x | x | x | x | 18 | 14 | 10 | 8 | 6 |
| 0.100 | – | – | x | x | x | 11 | 9 | 7 | 5 |




(a) $R = 0.1$ (b) $R = 0.2$ (c) $R = 0.3$ (d) $R = 0.5$

Fig. 7. Effect of the size of the robot on connectivity $P$

Fig.9 show the effect of the noise on $P$. For each $N$, $R$ of larger than 1.0 is required for that $P$ converges to 1.0. For large $N$, $P$ for uniform distribution is larger than the one for Gaussian distribution with the same value of $R$. For small $N$, $P$ for Gaussian distribution becomes large at relatively small $R$ compared to those for uniform distribution (Fig.9(b)). Over the range $10 \leq N \leq 20$, $R$ of larger than 20.0 is required in the case of uniform distribution (the worse case) for that $P$ converges to 1.0. This means that the communication range, $R$, must be 20 times as large as the environment. From these results, we can say that the effective communication range under uncertainty is much less than the communication range of a transmitter.

## IV. DISCUSSION

### A. Utilization of the Results

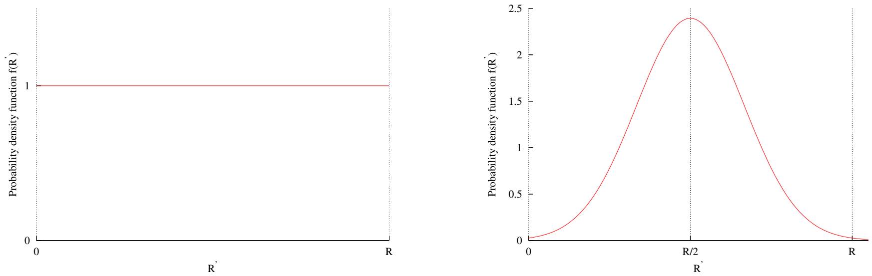
In this subsection, we propose the way to utilize the results obtained in the previous section. For the experimental setup in the previous section, the robots with communication range $R$ were randomly deployed in a unit square. Therefore, the width and the height are considered as 1.0, respectively and $R$ as a relative value for them.

Let communication range 200m in line-of-sight communication. For example, for a two-dimensional square with edges of length 400m, the communication range 200m corresponds to $R = 0.5$. Thus, 50 robots are required for connectivity according to Table I. For another example, a two-dimensional square with edges of length 250m corresponds to $R = 0.8$. Thus, 16 robots are required for connectivity.

When $R$ is large, we can decrease the number of robots required for connectivity by considering the body of the robot as mentioned in the previous section. In the above case where the edge of length is 250m and $R = 0.8$, $r = 250\text{m} \times 0.05 = 12.5\text{m}$. This results in a robot with a diameter of 25m. But this body length is not practical. When we assume typical communication range of a transmitter for WSN (100m to 200m) and a robot with the radius of less than 1m, $r$ becomes very small compared with $R$. This means that the results for $r = 0$ is sufficient for robots with practical volume.

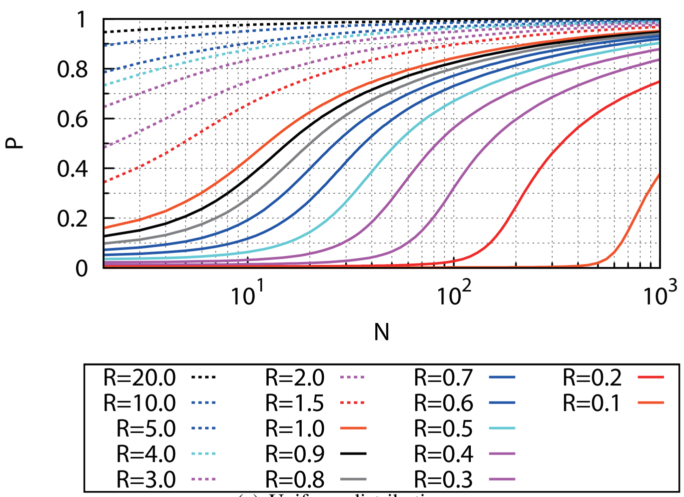
Under uncertainty, $R$ of larger than 1.0 is required for connectivity (Section III-B.3). Over the range $10 \leq N \leq 20$, $R$ must be 20 times as large as the environment. For these cases, we suppose that the environment can be divided into several regions. This results in that we distribute $N$ required for each smaller region. In the case of an unconvex region, e.g., an L-shaped or U-shaped corridor, we can apply this way.

For typical communication range, we have a problem that how much two-dimensional square we consider corresponding $R$ in Table I and the results obtained in Section III-B.3. These are decided due to the number of robots preparable by users and the control task to be solved by the SR.

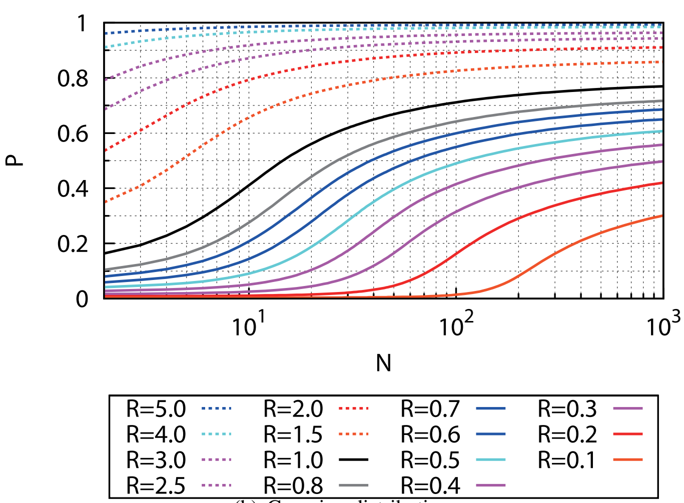



(a) Uniform distribution (b) Gaussian distribution

Fig. 8. Probabilistic communication model




(a) Uniform distribution




(b) Gaussian distribution

Fig. 9. Effect of the noise of the robot on connectivity $P$

### B. *Validity of Connectivity*

In this subsection, we investigate the validity of the results obtained in Section III. We conducted additional computer simulations where robots with the body in SR perform random walk with obstacle avoidance.

A two-wheeled robot was used in this experiment. The environment of the robot was a rectangular arena surrounded by walls with a base station placed at the bottom left corner (Fig. 10). The control task for a robot used in this experiment was navigating the environment where a robot avoids other robots. The robot was provided with 6 infrared proximity sensors which have a limited detection range in the

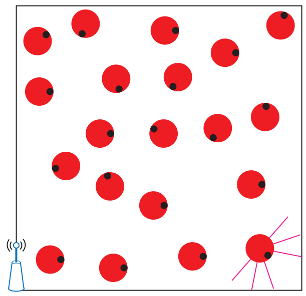
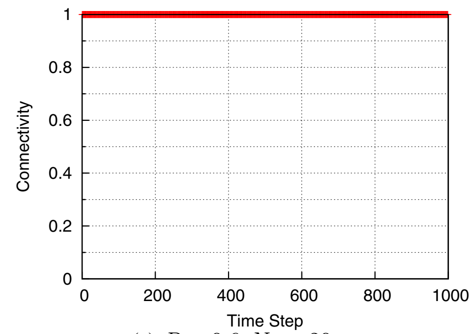


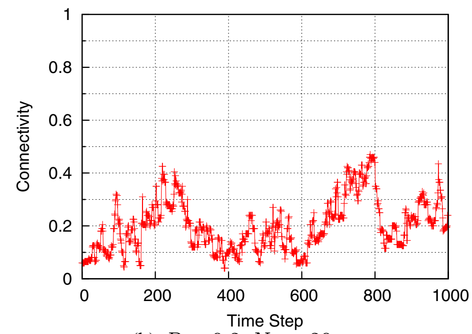
Fig. 10. Experimental setup for navigation problem

environment. If other robots or wall intersects a proximity sensor, the sensor outputs a value inversely proportional to the distance between the other robot or wall and the sensor. Employing a mathematical model of a mobile robot, the displacement of the robot was computed. At the beginning of each trial, robots were placed at random initial positions avoiding overlap at random orientations. According to Table I, the parameters were set as follows: The body of radius, $r$, is set at 0.05 and $(R, N_s) = (0.6, 20), (0.2, 20), (0.3, 50)$. One trial ends when 1000 time steps are performed. We conducted 10 independent runs. All results were averaged over 10 runs.

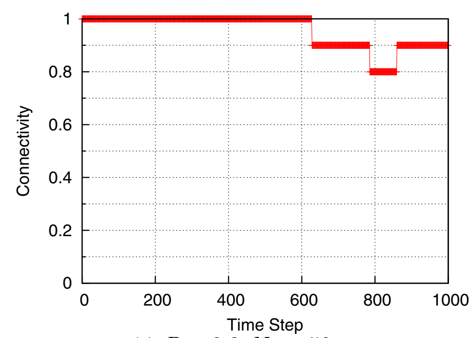
Figs. 11 show connectivity averaged in 10 runs for each time step. For $R = 0.6, N_s = 20$, connectivity were observable with relatively small number of robots over the runs because the environment were covered with wide communication range (Fig. 11(a)). When $R$ is small with the same $N_s$ ($R = 0.2, N_s = 20$), on the other hand, connectivity were not observable most over the run. For small $R$ and large $N_s$ ($R = 0.3, N_s = 50$), connectivity were observable most over the run. In several runs where connectivity were not kept in the last several hundred steps, about 10% robots came to a deadlock at the corners of the environment or among robots. This means that the robots were not distributed appropriately all over the environment.




(a) $R = 0.6, N_s = 20$




(b) $R = 0.2, N_s = 20$




(c) $R = 0.3, N_s = 50$

Fig. 11. Connectivity for each time step

## V. CONCLUSIONS AND FUTURE WORKS

This paper investigated communication range and the number of robots required for swarm robot networks to achieve connectivity based on percolation theory. Critical values for them were calculated under various conditions. Based on these results, it was confirmed in computer simulations that most mobile robots can navigate the environment keeping connectivity. Moreover, utilization of the results obtained in a series of simulation was discussed.

In real environment, quality of wireless communication maybe deteriorate due to several causes, e.g., obstacles in environment, interference of other transmissions or traffic congestion in wireless networks. In this study, we introduced the probabilistic communication model. In future works, we will investigate the validity of this model in real environment. Addition to this, we plan behavior design for real robots to keep connectivity of the network in the control task.

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