Estimating the Degree of Neutrality in Fitness Landscapes by the Nei's Standard Genetic Distance
– An Application to Evolutionary Robotics –

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Abstract— **In recent years, not only ruggedness but also neutrality has been recognized as an important feature of a fitness landscape for genetic search. As it has been reported that the evolutionary dynamics on a fitness landscape with neutrality is clearly different from the canonical explanations, ruggedness alone might be inadequate describing it. Another measure, i.e., neutrality is required. In this paper, we proposed the use of the *Nei's standard genetic distance*, which originates from population genetics, for estimating the degree of neutrality in fitness landscapes after minor modifications. Several computer simulations were conducted with an evolutionary robotics problem in order to investigate the validity of the proposed approach. The results suggest to us that the Nei's genetic distance is a reliable method for estimating the degree of neutrality on real-world problems.**

I. INTRODUCTION

In early works of the theoretical GA community, problem difficulties for a GA have been discussed in terms of the geography of a fitness landscape: *isolation*, *deception* and *multimodality*. Their factors affecting the performance of the GA to solve optimization problems are still contentious issues. However, several counterexamples on the latter two factors were found, showing that they are neither necessary nor sufficient to make a problem difficult [1][2][3].

Another attempt to characterize difficulty has been done by measuring the feature of a fitness landscape, *epistasis* or *ruggedness*. Most of the works in this area are based on the correlation between a parent fitness and the offspring one to describe a fitness landscape [1][4][5][6][7][8]. Therefore, it is derived from the average of fitness correlations between parents and offspring or the fitness distance autocorrelation function obtained by using a random walk. In the GA community, the majority of fitness landscape descriptions have based on ruggedness.

In recent years, the existence of problem domains has been reported where evolutionary dynamics is clearly different from the canonical explanations based on the schema theory and the building block hypothesis [9][10]. Those kinds of problem, such as the evolution of neural network controllers in robotics [11][12][13][14] and on-chip electronic circuit evolution [15][16][17], mainly show equilibrium period, *neutral evolution* in their evolutionary dynamics. This characteristic is caused by highly redundant mappings from genotype to phenotype or phenotype to fitness. With these kinds of problems, redundancy is inevitable although it is customary among GA practioners deliberately to avoid redundancy in the genetic coding of artificial evolution problems. Therefore, ruggedness alone is not enough to measure the search difficulty if equilibrium periods seem dominative in the process of evolution.

To the best of our knowledge, statistical measurements with respect to neutrality are found only in the references [16][18]. Vassilev *et al.*, studied the structure of on-chip electronic circuit evolution landscapes [16]. They proposed the *information analysis* of fitness landscapes, which is defined over a time series obtained by a walk on a landscape. Their entropic measure of the time series makes it possible to confirm the existence of neutrality in a landscape and which feature, neutrality or ruggedness is dominant in the landscape. Smith *et al.*, [18] proposed a method for measuring neutrality in a landscape as one of the *fitness evolvability portraits*. This is defined as the probability that an offspring fitness is equal to the parent fitness. In their measurement, a certain distinctive difference, ε, between two fitness values considered to be neutral must be set by GA practioners, that is, $|f_x - f_y| \leq \varepsilon$ (f_x is a parent fitness and f_y is the offspring one). However, the reference [16] has reported that Smith's measure is very sensitive to ε. Especially in the case that the fitness is evaluated as a real value or in a noisy environment, a great influence of the value on the measure of neutrality would be predicted. For these problems, Smith proposed the use of *neutral fitness band* [12] or the significance level for the Student t-test [19] as the value. However, no significant difference between two fitness landscapes was detected although introducing such statistical neutrality [19]. This implies the difficulty to use fitness data for measuring neutrality.

Population geneticists have been trying to explain the change of gene frequency in a population. That is, they have used genotype data for their explanations. Recently, we have been motivated by this to investigate the characteristics of the Nei's standard genetic distance[20], which is one of statistical methods for estimating gene differences between populations in population genetics, in artificial evolution [21]. The results show some consistencies with the neutral theory [22] and the nearly neutral theory [23][24] in population genetics.

The characteristics of the Nei's standard genetic distance can be summarized as follows:

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When the mutation rate per locus is sufficiently small,

1) The genetic distance increases approximately linearly over generations in fitness landscapes with neutrality.
2) The genetic distance increases with the increase of neutrality.
3) The genetic distance decreases with the increase of ruggedness in landscapes with neutrality.
4) The genetic distance decreases with the increase of the population size.

From the viewpoint of measuring neutrality, the characteristics 1) and 2) show the possibility of a genetic distance as an index of neutrality, while the characteristics 3) shows the difficulty to apply the genetic distance directly to the measure of neutrality.

This paper investigates how well the Nei's genetic distance applies to estimate the degree of neutrality in fitness landscapes. The greatest advantage of our approach is that the difficulty to use fitness data for measuring neutrality, which was mentioned above, need not to be taken into account because it uses genotype data for it. The paper is organized as follows. The next section describes the Nei's standard genetic distance. Section III applies the Nei's genetic distance to tunably neutral landscapes and shows the characteristics of the genetic distance. Section IV shows some guidelines for estimating the degree of neutrality in a fitness landscape on a real-world problem as well as investigates the validity of the proposed approach on a robot control problem. Conclusions are given in the last section.

II. THE NEI'S STANDARD GENETIC DISTANCE

Genetic distance is a term of population genetics used for estimating gene differences per locus between populations. Although there are several definitions for this, the Nei's standard genetic distance [20] is adopted in this paper.

The Nei's standard genetic distance is defined as follows. Consider two populations, X and Y. Let $x_{ik} = n_{ik}/M$ and $y_{ik} = n_{ik}/M$ be the frequencies of the k-th alleles ($i = 1, \cdots, N$, N, the length of the genotype, $k \in \{1, 2\}$ in a binary coded GA, n_{ik}, the number of the k-th allele, M, the population size) in X and Y, respectively. The probability of identity of two randomly chosen genes is $j_{xi} = x_{i1}^2 + x_{i2}^2$ in the population X, while it is $j_{yi} = y_{i1}^2 + y_{i2}^2$ in the population Y. The probability of identity of a gene from X and a gene from Y is $j_{xyi} = x_{i1}y_{i1} + x_{i2}y_{i2}$. The normalized identity of genes between X and Y with respect to a locus is defined as

$$I_i = \frac{j_{xyi}}{\sqrt{j_{xi}}\sqrt{j_{yi}}}, \tag{1}$$

where, $I_i = 1.0$ if the two populations have the same alleles in identical frequencies, and $I_i = 0.0$ if they have no common alleles. The normalized identity of genes between X and Y with respect to the average in all loci is defined as

$$I = \frac{J_{XY}}{\sqrt{J_X}\sqrt{J_Y}}, \tag{2}$$

where, $J_X = \sum_{i=1}^{N} j_{xi}/N$, $J_Y = \sum_{i=1}^{N} j_{yi}/N$ and $J_{XY} = \sum_{i=1}^{N} j_{xyi}/N$. The genetic distance between X and Y is defined as

$$D = -\log_e I. \tag{3}$$

The above definition cannot be applied to the standard GA directly, because it is assumed that a new allele always appears on a locus when a mutation occurs, while "back mutations [23]" frequently occur in the standard GA, due to the binary coding scheme. Therefore, the genetic distance of GAs between the population at the initial generation and the one at the last generation is calculated as:

$$D(T) = \sum_{t=1}^{T-1} D_{t,t+1} \tag{4}$$

where T is the number of the last generation and $D_{t,t+1}$ is the genetic distance between the population in the t-th and the $(t+1)$-th generation.

III. THE NEI'S STANDARD GENETIC DISTANCE IN TUNABLY NEUTRAL NK LANDSCAPES

In this section, we apply the standard GA (SGA)[25] to tunably neutral landscapes and obtain the genotype data from it. Considering to apply to real-world problems, we investigate the characteristics of the Nei's genetic distance for ruggedness measured by a canonical method.

A. Terraced NK Landscapes

A terraced NK landscapes and an extended one were employed as test functions in our computer simulations. The former is the tunably neutral NK landscape (TNK) proposed by Newman and Engelhardt [26] and the latter is the TNKp landscape which we extended to increase neutrality of TNK.

A terraced NK landscape has three parameters: N, the length of the genotype; K, the number of epistatic linkages between genes; and v, the contribution of a locus to the fitness of the entire genotype. The fitness value is calculated as follows: The fitness contribution of the i-th locus, v_i, is an integer generated randomly in the range $0 \leq v_i < F$, $i = 1, \cdots, N$. To calculate the fitness, V, of a genotype, the fitness contribution of each locus is averaged, and then divided by $F - 1$, normalizing V to the range 0.0 to 1.0. More formally:

$$V = \frac{1}{N(F-1)} \sum_{i=1}^{N} v_i. \tag{5}$$

The neutrality of the landscape can be tuned by changing the value of F. The neutrality of the landscape is maximized when $F = 2$, and is effectively non-existent as $F \rightarrow \infty$.

TNKp is an extended form of TNK for $F = 2$ in order to increase neutrality of TNK. For TNKp, v_i in Equation (5) is set at 0 with the probability, P $(0 \leq P \leq 1)$, following the way to involve neutrality in NKp fitness landscapes [27].

B. Simulation Conditions

Computer simulations were conducted using populations of size 50 by varying the landscape parameters. We applied the SGA. The SGA used standard bit mutation as the genetic operation. The per-bit mutation rate, q, was set at 0.008, based on the assumption in Section I. Crossover was not employed, following Nimwegen's suggestion [10]. Tournament selection was adopted. The tournament size was set at 2 because the SGA generally prefers low selection pressure. A generational model was used. Each run lasted 2,000 generations. We conducted 50 independent runs for each problem under the landscape parameters, $N = 20$, $K \in \{0, 2, 6, 12, 19\}$, $F \in \{2, 3, 4\}$, $P \in \{0.9, 0.99\}$. The results were averaged over 50 runs. As mentioned in Section I, the genetic distance increases approximately linearly over generations in fitness landscapes with neutrality. Therefore, the gradient of the genetic distance over generations, α, is calculated as an index of increase of the genetic distance in this experiment by using the method of least squares on the results of all the runs.

C. Simulation Results

1) Measure of Ruggedness: In real-world problems, ruggedness of a fitness landscape is predicted by the fitness correlation [5][18]. In this paper, therefore, the Smith's measurement [18] is employed for the measure of ruggedness because its fitness correlation can be expressed as a scalar value. The average fitness of the offspring solutions, called the Smith's E_b, is given by

$$E_b(k) = \frac{\sum_{g \in G(k)} V(g)}{|G(k)|} \tag{6}$$

where, $G(k)$ is the set of offspring from parents with the fitness k, g is an offspring genotype and $V(.)$ is the fitness function. Figure 1 show E_bs in TNK for $F = 2$. It is found that the gradient, $\dot{E}_b$, decreases with the increase of K. It has been known that this gradient is independent of F (neutrality), that it is proportional to the autocorrelation function for the mutation operator applied, and that $\dot{E}_b \simeq 1.0$ when $K = 0$ and $\dot{E}_b \simeq 0.0$ when $K = N - 1$.

2) Characteristics of the Genetic Distance: Figure 2 shows α at each correlation, $\dot{E}_b$, corresponding to K with all Fs and Ps in TNK(p). It is confirmed that α increased with the increase of $\dot{E}_b$ for all Fs and Ps, and increased with the decrease of F and the increase of P for all $\dot{E}_b$s. These are consistent with the results obtained in [21]. As a result, a set of points $(\dot{E}_b, \alpha)$ forms a curve, which increases with the increase of the correlation when F and P are constant. A set of curves is also found with different levels of neutrality. This demonstrates that α would predict the increase of neutrality combined with the measure of ruggedness.

IV. ESTIMATING THE DEGREE OF NEUTRALITY IN EVOLUTIONARY ROBOTICS FITNESS LANDSCAPES

A. Guidelines for Estimating the Degree of Neutrality

The procedure for estimating the degree of neutrality in a fitness landscape on a real-world problem can be summarized as follows:

i) Confirm the existence of neutrality in a fitness landscape on a real-world problem by investigating transitions of the genetic distance over generations.
ii) Calculate a point, $(\dot{E}_b, \alpha)$, from runs on the real-world problem.
iii) Calculate a set of $\dot{E}_b$-α curves in test functions with different levels of neutrality and ruggedness.
iv) By using a set of $\dot{E}_b$-α curves as baselines, judge where a point, $(\dot{E}_b, \alpha)$, obtained from the real-world problem locates in the graph of $(\dot{E}_b, \alpha)$. Then estimate indirectly the degree of neutrality on the real-world problem.

The reason why the procedure iii) and iv) are conducted is that the genetic distance is affected by both neutrality and ruggedness as confirmed in Section III. TNK and TNKp are adopted as test functions for obtaining a set of $\dot{E}_b$-α curves. The landscape parameters of TNK(p) should be decided at the procedure iii) for conducting the procedure iv).

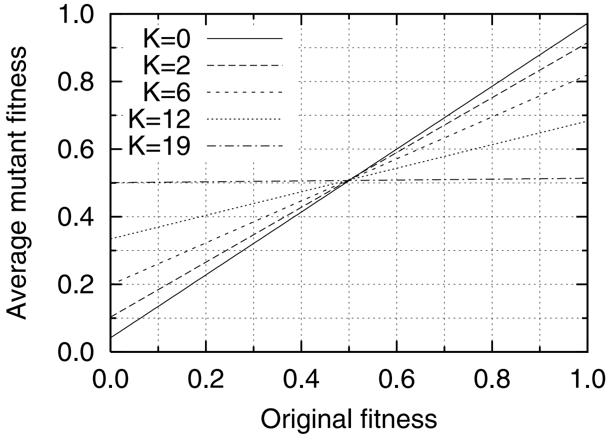

Fig. 1. Expected offspring fitness value over all parent fitness values in TNK for $F = 2$

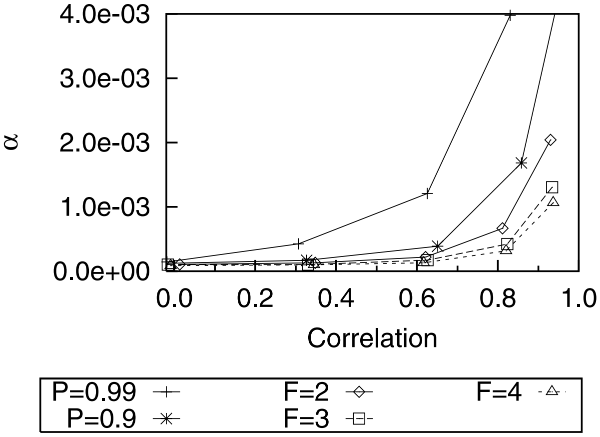

Fig. 2. α as a function of the correlation ($\dot{E}_b$)

Addition to this, the parameters of the GAs (a mutation rate, a population size, a tournament size, etc.) on test functions should be the same as those on a real-world problem due to the characteristics of the Nei's standard genetic distance.

B. The Task and the Fitness Function

The control task used in this experiment was motion pattern discrimination [28][29], and is based on a task originally implemented by Beer [30]. The agent must discriminate between two types of vertically falling object based on the object's period of horizontal oscillation; it must catch (i.e., move close to) falling objects that have a long period whilst avoiding those with a short period (see Fig. 3). An array of proximity sensors allow the agent to perceive the falling objects. If an object intersects a proximity sensor, the sensor outputs a value inversely proportional to the distance between the object and the agent. The agent can move horizontally along the bottom of the arena. In our experiment, the agent of diameter 30 had 7 proximity sensors of maximum range 220 uniformly distributed over a visual angle of 45 degrees. The horizontal velocity of the agent was proportional to the sum of the opposing horizontal forces produced by a pair of effectors. It has maximum velocity of 8. Each falling object was circular, with diameter 30, and dropped from the top of the arena with a vertical velocity of 4, a horizontal amplitude of 30 and an initial horizontal offset of ±50. An object's horizontal velocity was ±10 (12 steps in a period) for a long period and ±30 (4 steps in a period) for a short period.

The performance measure to be maximized was as follows:

$$Fitness = 1000 \sum_{i=1}^{NumTrials} \frac{H_i}{NumTrials} \tag{7}$$

where $H_i = 1 - d_i$ for a long period and $H_i = d_i$ for a short period, $d_i = 1$ when $hd_i > 60$ and $d_i = hd_i/60$ when $hd_i \leq 60$, hd_i is the final horizontal distance between the center of the agent and the object, and $NumTrials$ is the number of trials for an individual (8 trials for each period).

C. Simulation Conditions

For this experiment, the agent controller was a *spike response model* network [31], which is a form of *Pulsed Neural Network* (PNN) with 7 sensory neurons, 2 fully interconnected motor neurons and N_h fully interconnected hidden neurons, where $N_h \in \{0, 1, 5, 10, 15\}$ in order to estimate and compare the features among the fitness landscapes with each N_h. The network's connection weights and the firing threshold for each neuron were genetically encoded and evolved. The total number of parameters is equal to $\{20, 33, 105, 240, 425\}$ corresponding to each N_h. The parameters were mapped linearly with the following ranges: connection weights $\omega \in [-1.0, 1.0]$, thresholds $\theta \in [0.0, 3.9]$. The parameters of the neurons and synapses were set as follows: $\tau_m = 4$, $\tau_s = 10$, $\Delta^{ax} = 2$ for all neurons and all synapses in the network following the recommendations given in [32].

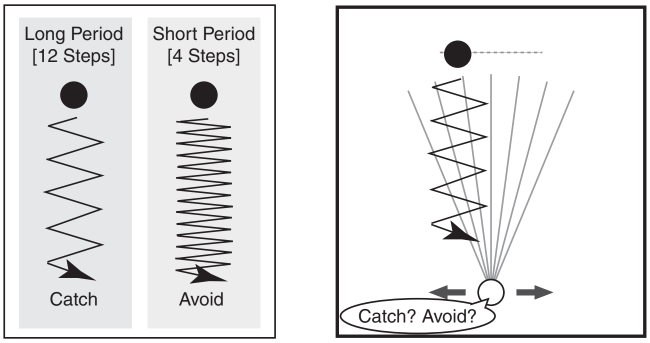

Fig. 3. Experimental setup for the discrimination of the motion patterns. Two kinds of period used in the discrimination experiments (left) and the agent in the arena with its array of the proximity sensors (right).

Computer simulations were conducted using populations of size 50. Each individual was encoded as a binary string with 10 bits for each parameter. Therefore, the total length of the genotype was $L = \{200, 330, 1050, 2400, 4250\}$ for each N_h. The SGA were adopted to evolve PNN parameters. The genetic operation for the SGA was standard bit mutation. Based on the assumption of the Nei's genetic distance in Section I, two types of mutation rate were set as follows:

a) $q = 1/L_{N_h=15}$, which is constant for each landscape, corresponding to $1/L$ for the longest genotype.
b) $q = 1/L$, following the recommendation in the evolutionary computation community.

According to the procedures in the previous subsection, all the parameters of the SGA must be the same among fitness landscapes to compare the features. Thus, the other parameters were set as follows. Tournament selection was adopted. Elitism was applied. The tournament size was set at 2. Each run lasted 6,000 generations. We conducted 10 independent runs for each landscape.

As the test functions, TNK and TNKp were adopted. A set of landscape parameters of TNK(p) at the procedure iii) was decided in the preliminary computer simulations with the same SGA parameters as those of the PNNs. In this experiment, they were set at the same values as those of TNK(p) in Section III. The reason to conduct the procedure iv) in these conditions is explained as follows: In the case of **a)** there are cases in which it is not possible to estimate and compare the degree of neutrality between fitness landscapes even if combined with the measure of ruggedness (will be described in the next subsection), and in the case of **b)** there is no theoretical meaning to compare directly the features of the fitness landscape of the PNNs due to the different mutation rates.

D. Simulation Results

a) $q = 1/L_{N_h=15}$

Figure 4 shows the genetic distance at each generation for $N_h = 15$. The approximately linear increases were observed in all runs. For other N_hs, the same transition were observed. From the results obtained in [21], this might indicate the presence of neutrality in the fitness landscape of the PNN.

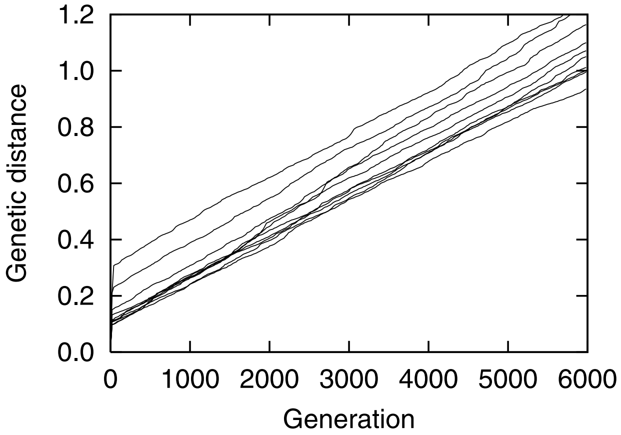

Fig. 4. Genetic distance at each generation for the SGA with $q = 1/L$ for $N_h = 15$ in 10 runs

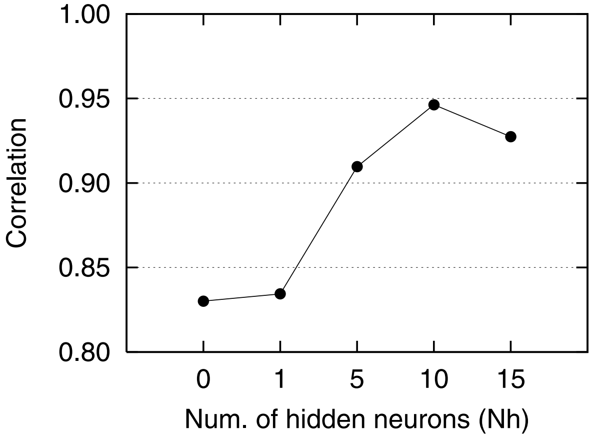

Fig. 5. Correlation ($\dot{E}_b$) for each N_h at $q = 1/L_{N_h=15}$

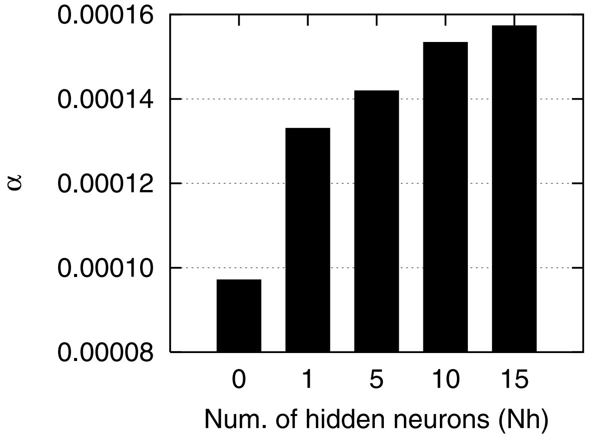

Fig. 6. α for each N_h at $q = 1/L_{N_h=15}$

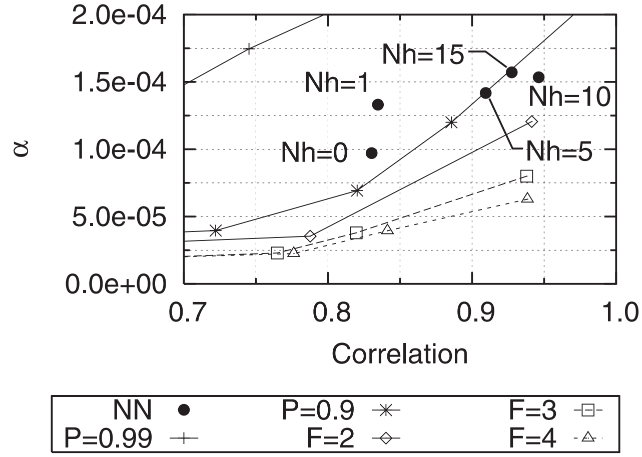

Fig. 7. α as a function of the correlation ($\dot{E}_b$) for each N_h and TNK(p) at $q = 1/L_{N_h=15}$

Thus, the gradient of the genetic distance over generations, α, is shown by using the method of least squares on the results of all the runs in the following parts.

Figure 5, 6 and 7 show the features of the fitness landscape for each N_h. Figure 5 plots the correlation, $\dot{E}_b$, for each N_h. In this experiment, the correlation increased with the increase of N_h except of $N_h = 15$. That is, ruggedness decreased with the increase of N_h. In Figure 6, α increased with the increase of N_h. When the fitness landscape for $N_h = 1$ was compared with the one for $N_h = 0$, there was no significant differences in the correlation but α increased so much. For another instance, when the fitness landscape for $N_h = 15$ was compared with the one for $N_h = 10$, the correlation decreased but α increased. In these cases, the increase of neutrality can be estimated based on the results obtained in [21]. Note that the increase of neutrality cannot be estimated either when the fitness landscape for $N_h = 5$ was compared with the one for $N_h = 1$ or when the fitness landscape for $N_h = 10$ was compared with the one for $N_h = 5$, because the increase of α coincided with the increase of the correlation.

By using the $\dot{E}_b$-α curves obtained in TNK(p), where F and P are constant for each curve, as baselines, the degree of neutrality in the landscapes for all N_hs were estimated indirectly (Figure 7). The points, $(\dot{E}_b, \alpha)$, for $N_h = 0$ and $N_h = 1$ locate between the $\dot{E}_b$-α curves for $P = 0.99$ and $P = 0.9$. The point for $N_h = 1$ is nearer to the curve for $P = 0.99$ than the point for $N_h = 0$. The points for $N_h = 5$ and $N_h = 15$ locate on the curve for $P = 0.9$. The points for $N_h = 10$ locates between the the curves for $P = 0.9$ and $F = 2$. Therefore, we confirmed that neutrality increased in order of $N_h = 10 \rightarrow \{5, 15\} \rightarrow 0 \rightarrow 1$.

In the cases of $N_h = 0 \rightarrow 1$, $10 \rightarrow 15$, the increases of neutrality were observed. These are consistent with the results of the direct comparison between the landscapes mentioned above. Additionally, the decreases of neutrality were also observed in the cases of $N_h = 1 \rightarrow 5$, $5 \rightarrow 10$ although it was not possible to estimate them by the direct comparison.

Figure 8 shows the maximum fitness at each generation for N_h. Except of $N_h = 1$, the fitness increased faster as the decrease of N_h, that is, the decrease of the genotypic search space. The poor performance in the SGA for $N_h = 1$ cannot be predicted by the comparison with $N_h = 0$ only on

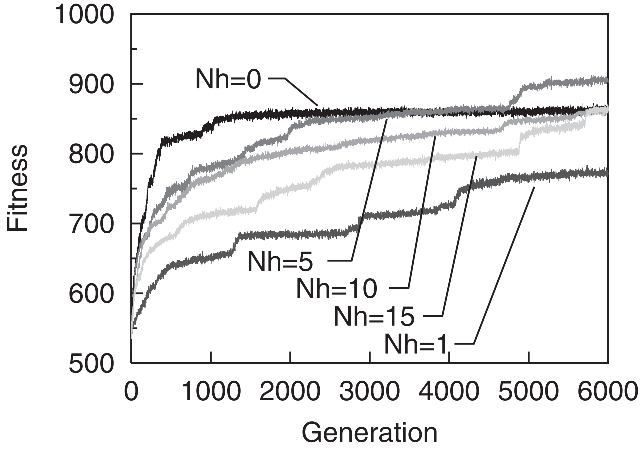

Fig. 8. Maximum fitness at $q = 1/L_{N_h=15}$

the correlation in Figure 5. But it would be predicted by the increase of neutrality observed in Figure 7 where equilibrium periods were more dominative in the process of evolution.

b) $q = 1/L$
Figure 9 plots the correlation, $\dot{E}_b$, for each N_h. As observed in **a)**, the correlation increased with the increase of N_h except of $N_h = 15$. Because of different q among the landscapes, the degree of neutrality in the landscapes for all N_hs can be estimated indirectly by using the $\dot{E}_b$-α curves obtained in TNK(p) (Figure 10). It was observed that neutrality increased in order of $N_h = 10 \rightarrow 5 \rightarrow \{1, 15\} \rightarrow 0$.

Figure 11 shows the maximum fitness at each generation for N_h. The fitness increased faster as the decrease of N_h, that is, the decrease of the genotypic search space. This would be explained as follows; In the process of evolution, no error threshold effects were observed [1]. This implies that the effective mutation rate at $q = 1/L$ would be below the error threshold under each condition [29][34][35]. In addition, ruggedness were not so high in this problem (Figure 9) Therefore, neutrality of the landscapes would not explicitly have any influences on the performances of the SGA but the size of the genotypic search space would have influences on them.

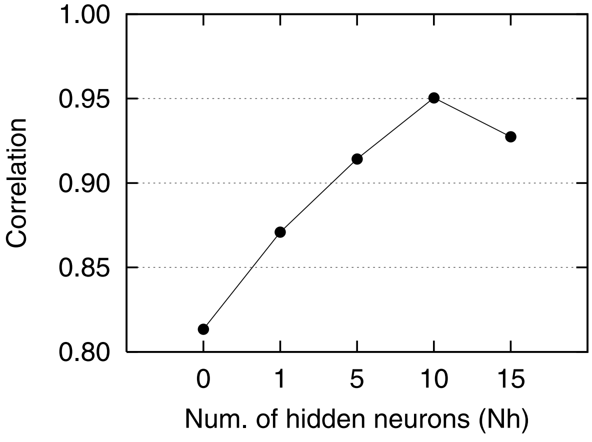

Fig. 9. Correlation ($\dot{E}_b$) for each N_h at $q = 1/L$

From the viewpoint of the structure of evolutionary artificial neural networks, it was confirmed by the proposed approach that the number of hidden neurons have a great influence on the features of the fitness landscape of neural networks.

V. CONCLUSIONS

In this paper, we proposed the use of the Nei's standard genetic distance, which originates from population genetics, for estimating the degree of neutrality in fitness landscapes after minor modifications. We showed the guidelines to apply our approach to a real-world problem. Several computer simulations were conducted with an evolutionary robotics problem in order to investigate the validity of the proposed approach. Our results can be summarized as follows:

- The existence of neutrality in an evolutionary robotics fitness landscape was confirmed by investigating transitions of the genetic distance over generations.
- The proposed method can estimate and compare indirectly the degree of neutrality in the fitness landscapes by using the features of the fitness landscape of test functions as baselines.

These results suggest the validity of the proposed approach. By using this, we might expect to explain evolutionary dynamics on problems where they have not been explained by canonical fitness landscape descriptions.

This evolutionary robotics problem has redundant mappings from phenotype to fitness. Future work will investigate whether our approach is applicable to real-world problems which are expected to have redundant mappings from genotype to phenotype [36][37][38][39].

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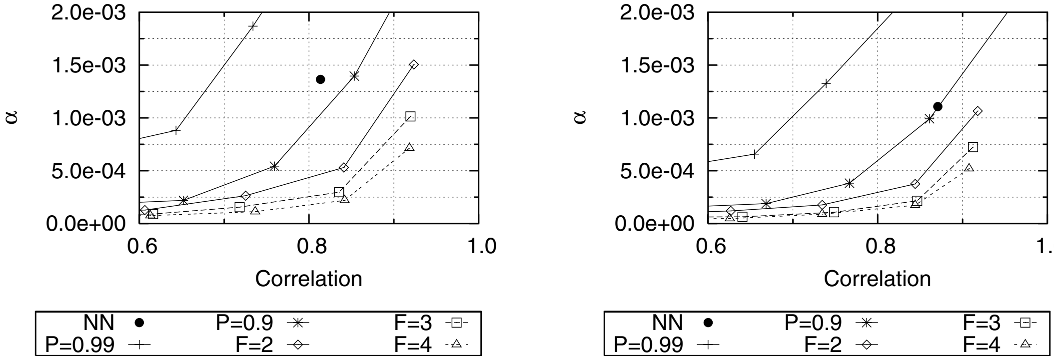
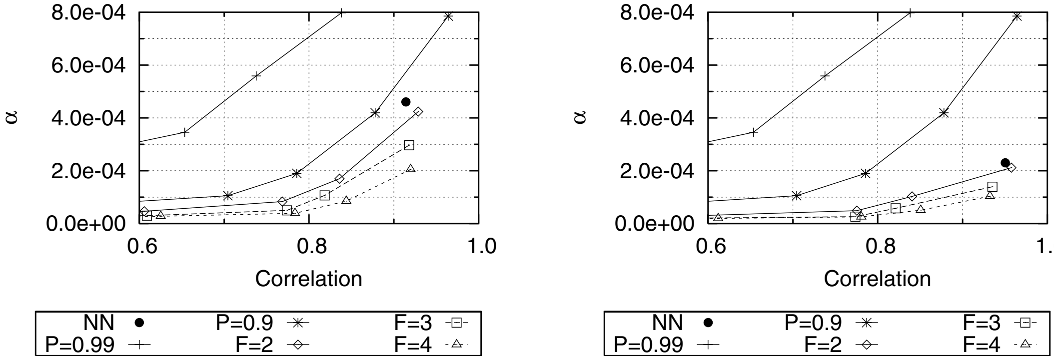
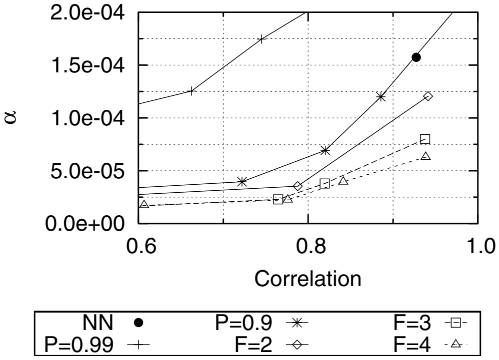
[1] Generally, an error threshold sets the upper limit for a mutation rate that will enable efficient search [10][27][33].



(a) $N_h = 0$ (b) $N_h = 1$



(c) $N_h = 5$ (d) $N_h = 10$



(e) $N_h = 15$

Fig. 10. α as a function of the correlation ($\dot{E_b}$) for each N_h and TNK(p) at $q = 1/L$

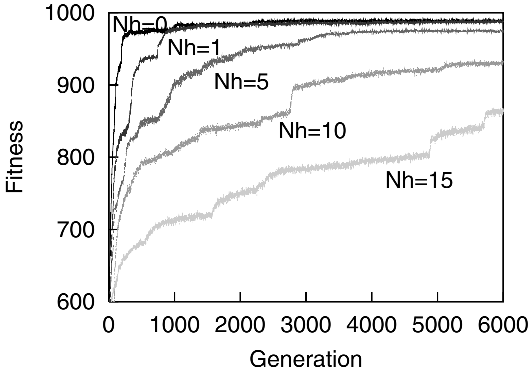


Fig. 11. Maximum fitness at $q = 1/L$

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